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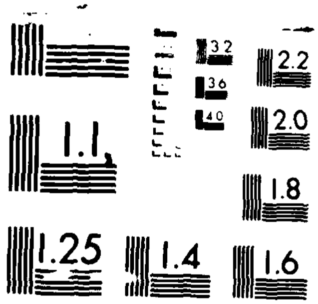


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# THE PMC SYSTEM LEVEL FAULT MODEL: CARDINALITY PROPERTIES OF THE IMPLIED FAULTY SETS

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REPORT JHU/ECE-87/06

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This work was supported by the Air Force Office of Scientific Research under Contract AFOSR-85-0097.



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## ABSTRACT

In this paper, we consider one aspect of the PMC system level fault model, the properties of the implied faulty sets. For $r$-diagnosable systems that have at most $r$ faulty units, we give lower bounds on the cardinality of the maximal implied faulty sets, then we show that these bounds are greatest lower bounds and we indicate how these results may be used in diagnosis algorithms.

# I. INTRODUCTION

The PMC system level fault model [PRE67] consists of a set of units $U = \{u_1, u_2, \ldots, u_n\}$ capable of testing one another and a set of ordered pairs $\{(u_i, u_j) \mid u_i \textit{ tests } u_j\}$ describing the organization of the tests. The model is defined by the fault-test relationship which specifies the test outcome $a_{i,j}$ in terms of the status of both the unit $u_i$ applying the test and the unit $u_j$ being tested. If $u_i$ is nonfaulty, then $a_{i,j} = 0$ if $u_j$ is nonfaulty and $a_{i,j} = 1$ if $u_j$ is faulty, and if $u_i$ is faulty, the test outcome $a_{i,j} = 0$ *or* 1, independent of the status of $u_j$. A collection of all test outcomes is called a syndrome. The model can be represented by the directed graph $G = (U,E)$, in which the vertices in $U$ are the units and the edges in $E$ are the tests between units. The test outcomes are the edge labels of the graph, and thus $G$ has both 0-edges and 1-edges. The model has been studied extensively and among topics that have been addressed are conditions for $\tau$-diagnosability ([PRE67], [HAK74], [ALL75], [CHW81], [KEN84]) and algorithms for system diagnosis ([KAM75], [MEY78], [MAD77], [MEY81], [DAH84], [DAH85]).

Given a syndrome, the diagnosis problem consists of identifying the set of faulty units $F_S$ and the set of nonfaulty units $G_S$. A system is $\tau$-diagnosable if and only if all faulty units can be identified from the syndrome whenever the system has at most $\tau$ faulty units [PRE67]. For a given syndrome, a partition $(G,F)$ is consistent with the syndrome if every test among units in $G$ has a 0 outcome and every test from a unit in $G$ to a unit in $F$ has a 1 outcome. Diagnosis of a $\tau$-diagnosable system with at most $\tau$ faulty units requires identifying the unique consistent partition $(G_S, F_S)$ such that $\|F_S\| \le \tau$.

We recall that for a given syndrome, the implied nonfaulty set $G(u_i)$ for the unit $u_i$ is the set of all units that are implied nonfaulty if $u_i$ is assumed to be nonfaulty and

the implied faulty set $L(u_i)$ is the set of all units that are implied faulty if $u_i$ is assumed to be nonfaulty [KAM75]. Thus, if we define a *0-path* in the graph $G$ as a path in which every edge is a 0-edge, we see that

$$G(u_i) = \{u_i\} \cup \{u_j \mid \textit{there is a 0-path from } u_i \textit{ to } u_j\},$$

and

$$L(u_i) = \{u_j \mid \textit{there exists } u_p \textit{ in } G(u_i),\ u_q \textit{ in } G(u_j) \text{ and } \textit{either } a_{p,q} = 1 \textit{ or } a_{q,p} = 1 \textit{ or both}\}.$$

It is clear that if $L(u_i) \cap G(u_i) \neq \phi$, then the unit $u_i$ is faulty. Many diagnosis algorithms take advantage of this fact by declaring such units faulty and concentrating on the problem of diagnosing the resulting reduced system. For example, if a system is $\tau$-diagnosable and has at most $\tau$ faulty units, the algorithm in [MEY81] identifies the set of faulty units if there exists at least one faulty unit $u_i$ such that either $L(u_i) \cap G(u_i) \neq \phi$ or $\|L(u_i)\| \geq \tau + 1$. Only $\tau$-diagnosable systems in which no two units test each other are known to have this property [MAD77], [MEY83]. The structural constraints associated with self-implicating systems [DAH85] are even stronger.

In this paper we do not impose structural constraints on the test organization, and we analyze the properties of the implied faulty sets only under the assumptions that the system is $\tau$-diagnosable and that the number of faulty units is not greater than $\tau$. The main thrust of our effort is directed at obtaining lower bounds on the cardinality of the maximal implied faulty sets associated with not only the units in $F_S$, but also the units in $G_S$. We then present a brief example of how these bounds can simplify diagnosis. All proofs for this paper are contained in [KEN86].

## II. IMPLIED FAULTY SETS OF FAULTY UNITS

When $\tau \leq 2$, the cardinality of the maximal implied faulty sets associated with the

faulty units can be obtained without much difficulty.

*Theorem 1:* If $S$ is $\tau$-diagnosable, if $1 \leq \|F_S\| \leq \tau$, and if $\tau \leq 2$, at least one unit $u_i$ in $F_S$ exists such that $\|L(u_i)\| \geq \tau + 1$.

The next result shows that for the implied faulty sets associated with faulty units this lower bound is actually the greatest lower bound.

*Lemma 1:* To the integers $\tau = 1$ and $\tau = 2$ correspond at least one $\tau$-diagnosable system $S$ that has $\tau$ faulty units and one syndrome such that:

(i) $L(u_i) \cap G(u_i) = \phi$ for every unit $u_i$ in $S$,

(ii) $\|L(u_i)\| = \tau + 1$ for every faulty unit $u_i$, and

(iii) $\|L(u_i)\| = \tau$ for every nonfaulty unit $u_i$.

When $\tau > 2$, we obtain the following results.

*Theorem 2:* If $S$ is $\tau$-diagnosable, if $1 \leq \|F_S\| \leq \tau$, and if $\tau > 2$, at least one unit $u_i$ in $F_S$ exists such that either $L(u_i) \cap G(u_i) \neq \phi$ or $\|L(u_i)\| \geq \tau - k + 1$, where $k$ is the least integer such that $\tau \leq 6k + 2$.

The next result shows that for $\tau > 2$ the lower bound given in Theorem 2 is actually the greatest lower bound on the cardinality of the maximal $L(u_i)$ associated with the faulty units.

*Lemma 2:* To every integer $\tau > 2$ corresponds at least one $\tau$-diagnosable system $S$ that has $\tau$ faulty units and one syndrome such that:

(i) $L(u_i) \cap G(u_i) = \phi$ for every unit $u_i$ in $S$,

(ii) $\|L(u_i)\| = \tau - k + 1$ for every faulty unit $u_i$, where $k$ is the least integer such that $\tau \leq 6k + 2$, and

(iii) $\|L(u_i)\| = \tau$ for at least one nonfaulty unit $u_i$.

Theorems 1 and 2 show that the set of values of $\tau$ may be partitioned into intervals of length 6, except for the first interval that is of length 2. For $\tau$-diagnosable

systems in which both $1 \leq ||F_S|| \leq \tau$ and $L(u_i) \cap G(u_i) = \phi$ for all $u_i$ in $S$, Theorem 1 implies that if $\tau \leq 2$, at least one faulty unit $u_i$ exists such that $||L(u_i)|| \geq \tau + 1$, and Theorem 2 implies that if $\tau \leq 8$, at least one faulty unit $u_i$ exists such that $||L(u_i)|| \geq \tau$, if $\tau \leq 14$, at least one faulty unit $u_i$ exists such that $||L(u_i)|| \geq \tau - 1$, and so forth.

## III. IMPLIED FAULTY SETS OF ALL UNITS

It is clear that $||L(u_i)|| \leq \tau$ whenever the unit $u_i$ in nonfaulty, and therefore when $\tau \leq 2$, the consideration of nonfaulty units does not result in an improvement of the lower bound on the cardinality of $L(u_i)$.

*Theorem 3:* If $S$ is $\tau$-diagnosable, if $1 \leq ||F_S|| \leq \tau$, and if $\tau \leq 2$, at least one unit $u_i$ in $S$ exists such that $||L(u_i)|| \geq \tau + 1$.

When $\tau > 2$, Lemma 6 not only shows that the lower bound given in Theorem 2 is the greatest lower bound, but also that the unit with the maximal implied faulty set may be nonfaulty. We now improve the lower bound on the cardinality of the maximal $L(u_i)$ by considering not only the implied faulty sets associated with the faulty units, but also the implied faulty sets associated with the nonfaulty units.

The following theorem extends Theorem 3 by considering the implied faulty sets of both faulty and nonfaulty units.

*Theorem 4:* If $S$ is $\tau$-diagnosable, if $1 \leq ||F_s|| \leq \tau$, and if $\tau > 2$, at least one unit in $S$ exists such that either $L(u_i) \cap G(u_i) \neq \phi$ or $||L(u_i)|| \geq \tau - k + 1$, where $k$ is the least integer such that $\tau \leq 7k + 2$.

Lemma 6 shows that for $3 \leq \tau \leq 8$ the lower bound on the cardinality of the maximal implied faulty set given in Theorem 2 is the greatest lower bound. The next lemma proves a similar result for $\tau > 8$.

*Lemma 3:* To every integer $\tau > 8$ corresponds at least one $\tau$-diagnosable system $S$

that has $\tau$ faulty units and one syndrome such that:

(i) $L(u_i) \cap G(u_i) = \phi$ for every $u_i$ in $S$,

(ii) $||L(u_i)|| \leq \tau - k + 1$ for every $u_i$ in $S$, where $k$ is the least integer such that $\tau \leq 7k + 2$,

(iii) $||L(u_i)|| = \tau - k + 1$ for at least one faulty unit $u_i$, and

(iv) $||L(u_i)|| = \tau - k + 1$ for at least one nonfaulty unit $u_i$.

Theorems 3 and 4 show that the set of values of $\tau$ may be partitioned into intervals of length 7, except for the first interval of length 2. Thus, for a $\tau$-diagnosable system in which $1 \leq ||F_s|| \leq \tau$ and $L(u_i) \cap G(u_i) = \phi$ for all $u_i$ in $S$, Theorem 3 implies that if $\tau \leq 2$, at least one unit $u_i$ exists such that $||L(u_i)|| \geq \tau + 1$, and Theorem 4 implies that if $\tau \leq 9$, at least one unit $u_i$ exists such that $||L(u_i)|| \geq \tau$, if $\tau \leq 16$, at least one unit $u_i$ exists such that $||L(u_i)|| \geq \tau - 1$, and so forth.

## IV. CONCLUSION

We have presented results concerning the properties of the implied faulty sets in the PMC system level fault model. Unlike previous work on implied faulty set properties, we made no structural properties assumptions, only that the system was $\tau$-diagnosable and had at most $\tau$ faulty units. The results are not only interesting in themselves, but also because of their implications in the diagnosis process.

Given a $\tau$-diagnosable system $S$ and the implied faulty and nonfaulty sets for each unit, we can identify the set $F_0 = \{u_i \mid L(u_i) \cap G(u_i) \neq \phi\}$. If $S$ has at most $\tau$ faulty units, then $||F_0|| \leq \tau$. In this case, removing from $S$ the units in $F_0$ and all tests involving these units produces a reduced system $(S - F_0)$ that is $(\tau - ||F_0||)$-diagnosable. The results of this paper outline the properties of the maximal implied faulty sets in the reduced system $(S - F_0)$. If $(\tau - ||F_0||) \leq 2$, then the units with the maximal $||L(u_i)||$ are faulty. If $3 \leq (\tau - ||F_0||) \leq 9$, then there exists at least one unit

$u_i$ such that $||L(u_i)|| \geq \tau$. If $||L(u_i)|| > \tau$, then $u_i$ is obviously faulty. When $||L(u_i)|| = \tau$, the implied faulty and nonfaulty sets are the basis of a consistent partition of $S$. If the remainder set $N(u_i) = S - (L(u_i) \cup G(u_i))$ contains no 1-edges then $\{G(u_i) \cup N(u_i), L(u_i)\}$ is a minimal consistent partition of $S$ such that $||L(u_i)|| = \tau$, and thus $u_i$ must be nonfaulty and $F_S = L(u_i)$. Also note that if $||L(u_i)|| = \tau$ and $N(u_i)$ contains at least one 1-edge, then any consistent partition $\{G,F\}$ of $S$ in which $u_i$ is in $G$ is such that $||F|| > \tau$, and thus $u_i$ is faulty.

As an example of a system that is easily diagnosed using this approach, consider the 6-diagnosable, 13 unit system proposed by Madden [MAD77]. Figure 1 shows the test outcome/incidence matrix $B$ of this system. The $i^{th}$ row and $j^{th}$ column element $b_{ij}$ is equal to the test outcome $a_{i,j}$ (0 or 1) if $u_i$ tests $u_j$ and $b_{ij}$ is equal to $x$ if $u_i$ does not test $u_j$. For each $i$ in $\{1,2,\ldots,13\}$, Figure 2 lists the indices of the units $u_j$ in the implied faulty set $L(u_i)$, in the implied nonfaulty set $G(u_i)$, and in the remainder set $N(u_i) = S - (L(u_i) \cup G(u_i))$. Note that $L(u_i) \cap G(u_i) = \phi$ for all units $u_i$, $i$ in $\{1,2,\ldots,13\}$. The units such that $||L(u_i)|| = 6$ are easily diagnosed. For example, the unit $u_8$ has $||L(u_8)|| = 6$ and $N(u_8) = \{u_7,u_{10},u_{11},u_{12},u_{13}\}$. The set $N(u_8)$ contains the 1-edge $(u_7,u_{10})$ among others, thus $u_8$ must be faulty. On the other hand, the unit $u_2$ has $||L(u_2)|| = 6$ and $N(u_2) = \{u_4\}$, and thus the set of faulty units is $F_S = L(u_2) = \{u_8,u_9,u_{10},u_{11},u_{12},u_{13}\}$.

Figure 1 - Test Outcome/Incidence Matrix

| | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | x | x | x | x | x | x | x | 1 | 1 | x | x | x | x |
| 2 | 0 | x | 0 | x | 0 | 0 | x | 1 | 1 | 1 | 1 | 1 | 1 |
| 3 | 0 | 0 | x | x | 0 | x | x | 1 | 1 | 1 | 1 | 1 | 1 |
| 4 | 0 | x | 0 | x | x | x | 0 | 1 | 1 | 1 | 1 | 1 | 1 |
| 5 | 0 | x | x | x | x | 0 | 0 | 1 | x | 1 | 1 | 1 | 1 |
| 6 | 0 | x | x | x | 0 | x | x | x | 1 | 1 | 1 | 1 | 1 |
| 7 | x | x | x | x | x | x | x | x | x | 1 | 1 | 1 | 1 |
| 8 | 1 | 1 | x | 1 | x | x | x | x | 0 | x | x | x | x |
| 9 | x | x | x | 1 | x | 1 | x | 0 | x | x | x | x | x |
| 10 | x | 1 | 1 | 1 | 1 | x | 1 | x | x | x | x | x | x |
| 11 | x | 1 | 1 | 1 | x | 1 | 1 | x | x | x | x | x | x |
| 12 | x | 1 | 1 | 1 | 1 | 1 | 1 | x | x | x | x | x | x |
| 13 | x | 1 | 1 | 1 | 1 | 1 | 1 | x | x | x | x | x | x |

Figure 2 - The indices of the units in $L(u_i)$, $G(u_i)$, and $N(u_i)$ for each $u_i$ in $S$

| $i$ | $\{j \mid u_j \in L(u_i)\}$ | $\{j \mid u_j \in G(u_i)\}$ | $\{j \mid u_j \in N(u_i)\}$ |
|---|---|---|---|
| 1 | $\{8,9\}$ | $\{1\}$ | $\{2,3,4,5,6,7,10,11,12,13\}$ |
| 2 | $\{8,9,10,11,12,13\}$ | $\{1,2,3,5,6,7\}$ | $\{4\}$ |
| 3 | $\{8,9,10,11,12,13\}$ | $\{1,2,3,5,6,7\}$ | $\{4\}$ |
| 4 | $\{8,9,10,11,12,13\}$ | $\{1,2,3,4,5,6,7\}$ | $\phi$ |
| 5 | $\{8,9,10,11,12,13\}$ | $\{1,5,6,7\}$ | $\{2,3,4\}$ |
| 6 | $\{8,9,10,11,12,13\}$ | $\{1,5,6,7\}$ | $\{2,3,4\}$ |
| 7 | $\{10,11,12,13\}$ | $\{7\}$ | $\{1,2,3,4,5,6,8,9\}$ |
| 8 | $\{1,2,3,4,5,6\}$ | $\{8,9\}$ | $\{7,10,11,12,13\}$ |
| 9 | $\{1,2,3,4,5,6\}$ | $\{8,9\}$ | $\{7,10,11,12,13\}$ |
| 10 | $\{2,3,4,5,6,7\}$ | $\{10\}$ | $\{1,8,9,11,12,13\}$ |
| 11 | $\{2,3,4,5,6,7\}$ | $\{11\}$ | $\{1,8,9,10,12,13\}$ |
| 12 | $\{2,3,4,5,6,7\}$ | $\{12\}$ | $\{1,8,9,10,11,13\}$ |
| 13 | $\{2,3,4,5,6,7\}$ | $\{13\}$ | $\{1,8,9,10,11,12\}$ |

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Unclassified

SECURITY CLASSIFICATION OF THIS PAGE

# REPORT DOCUMENTATION PAGE

| | |
|---|---|
| 1a. REPORT SECURITY CLASSIFICATION: Unclassified | 1b. RESTRICTIVE MARKINGS |
| 2a. SECURITY CLASSIFICATION AUTHORITY | 3. DISTRIBUTION/AVAILABILITY OF REPORT: Unrestricted |
| 2b. DECLASSIFICATION/DOWNGRADING SCHEDULE | |
| 4. PERFORMING ORGANIZATION REPORT NUMBER(S): JHU/ECE-87/06 | 5. MONITORING ORGANIZATION REPORT NUMBER(S) |
| 6a. NAME OF PERFORMING ORGANIZATION: The Johns Hopkins University; 6b. OFFICE SYMBOL (If applicable) | 7a. NAME OF MONITORING ORGANIZATION: Air Force Office of Scientific Research /NM |
| 6c. ADDRESS (City, State and ZIP Code): Charles and 34th Streets, Baltimore, MD 21218 | 7b. ADDRESS (City, State and ZIP Code): Bolling AFB, Washington DC 20332 |
| 8a. NAME OF FUNDING/SPONSORING ORGANIZATION: AFOSR/PKZ; 8b. OFFICE SYMBOL (If applicable): N/A | 9. PROCUREMENT INSTRUMENT IDENTIFICATION NUMBER: AFOSR-85-0097 |
| 8c. ADDRESS (City, State and ZIP Code): Building 410, Bolling AFB - DC 20332-6448 | 10. SOURCE OF FUNDING NOS.: PROGRAM ELEMENT NO. / PROJECT NO. / TASK NO. / WORK UNIT NO |

11. TITLE (Include Security Classification) The PMC System Level Fault Model: Cardinality Properties of the Implied Faulty Sets

12. PERSONAL AUTHOR(S) Mary Ann Kennedy and Gerard G. L. Meyer

| 13a. TYPE OF REPORT | 13b. TIME COVERED | 14. DATE OF REPORT (Yr., Mo., Day) | 15. PAGE COUNT |
|---|---|---|---|
| Interim | FROM 1/1/87 TO 4/3/87 | 1987, April 4 | 12 |

16. SUPPLEMENTARY NOTATION: N/A

| 17. COSATI CODES | | | 18. SUBJECT TERMS (Continue on reverse if necessary and identify by block number) |
|---|---|---|---|
| FIELD | GROUP | SUB. GR. | Fault Model - Diagnosis - Faulty Sets |
| | | | |
| | | | |

19. ABSTRACT (Continue on reverse if necessary and identify by block number)


In this paper, we consider one aspect of the PMC system level fault model, the properties of the implied faulty sets. For $\tau$-diagnosable systems that have at most $\tau$ faulty units, we give lower bounds on the cardinality of the maximal implied faulty sets, then we show that these bounds are greatest lower bounds and we indicate how these results may be used in diagnosis algorithms.


| | |
|---|---|
| 20. DISTRIBUTION/AVAILABILITY OF ABSTRACT: UNCLASSIFIED/UNLIMITED ☒ SAME AS RPT. ☐ DTIC USERS ☐ | 21. ABSTRACT SECURITY CLASSIFICATION: Unclassified |
| 22a. NAME OF RESPONSIBLE INDIVIDUAL: Major Brian Woodruff | 22b. TELEPHONE NUMBER (Include Area Code): 202-767-5027; 22c. OFFICE SYMBOL |

DD FORM 1473, 83 APR  EDITION OF 1 JAN 73 IS OBSOLETE.

Unclassified

SECURITY CLASSIFICATION OF THIS PAGE